ANTIQUITATES MATHEMATICAE
Vol. 17, 2023, p. 163–188
doi: 10.14708/am.v17i1.7273

BIOGRAPHICAL NOTES

Lech Maligranda (Poznań and Luleå)
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Rubin (Robert) Schatten (1911-1977)

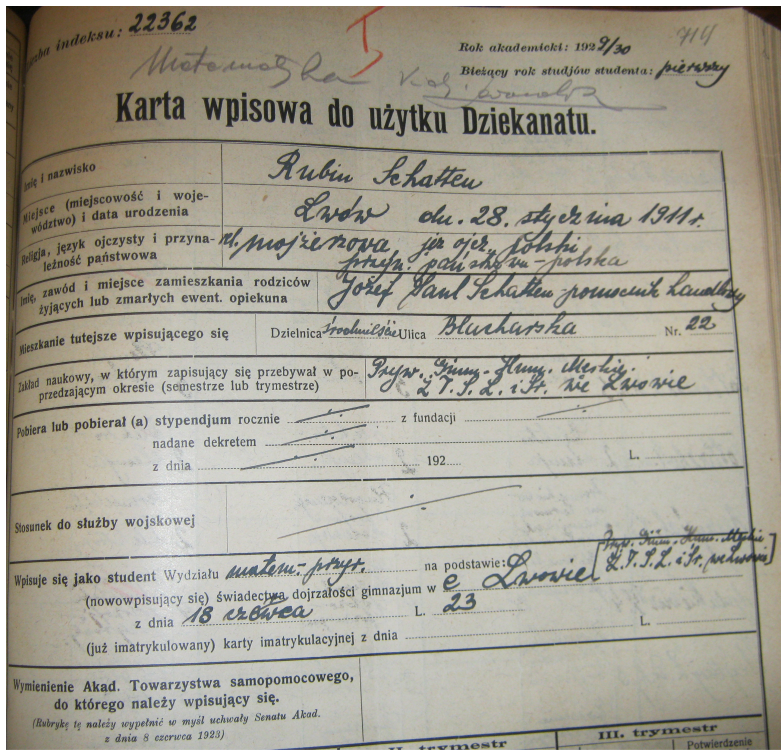
Abstract. *Rubin (Robert) Schatten* was a Polish-American mathematician, working mainly in functional analysis. In the years 1929-1933 he studied at the Jan Kazimierz University in Lwów, and a master degree has been written under supervision of Stefan Banach. From September 1936 until his death in August 1977 he stayed in the USA, where he obtained his doctorate in 1943 and worked at several American universities. Schatten has published 13 scientific papers and two monographs on operator theory. He was one of the pioneers of the systematic study of tensor products of Banach spaces. His name in mathematics was immortalized in Schatten's classes of operators. We became interested in Schatten because of the beginnings of his career in Lwów and the lack of a photo in all descriptions of him. Therefore, we present the course of his life. His photo will also appear for the first time.

2020 MSC: 01A60, 01A70, 01A73.

Key words and phrases: History of mathematics and mathematicians, Robert Schatten, functional analysis, tensor product, Schatten classes.

1. Biography. Rubin Schatten was born on 28 January 1911 in Lwów (then it was a Polish city under Austrian rule, now Lviv in Ukraine). The parents were Józef Saul Schatten (born on 28 March 1880 in Lwów, son of the merchant Samuel Schatten and Tradala née Rad) and Osne Laja Schatten née Sieder (born on 14 July 1885 in Niżankowice, a town 13 km south of Przemyśl, today in Ukraine, as the daughter of Chana Sieder and the talmudist Mojżesz Judy Sieder, a merchant and private teacher), who were married on 11 March 1910 in Lwów. At the time of Rubin's birth, the Schattens lived on Lwia Street No. 2. Information about the beginnings of Schatten's life comes from [Sc18]. We have not been able to establish anything more about his family, for example, we do not know if he had siblings. From 1942 he used the name Robert.

He graduated from the Private Male Humanistic Gymnasium of the Jewish People's and Secondary School Society in Lwów, receiving his matriculation certificate on 18 June 1929. In September 1929 he began studying at the Faculty of Mathematics and Natural Sciences at Jan Kazimierz University in Lwów (UJK). His index number was 22362. The photo of the first page of his entry card is presented on Fig. 1.


Liczba indeksu: 22362

Rok akademicki: 1929/30

Bieżący rok studjów studenta: pierwszy

Karta wpisowa do użytku Dziekanatu.

Imię i nazwisko: Rubin Schatten

Miejsce (miejscowość i województwo) i data urodzenia: Lwów dn. 28. stycznia 1911 r.

Religja, język ojczysty i przynależność państwowa: mojżeszowa, jęz. ojcz. polski, przyn. państw. polska

Imię, zawód i miejsce zamieszkania rodziców żyjących lub zmarłych ewent. opiekuna: Józef Paul Schatten pomocnik kancelaryjny

Mieszkanie tutejsze wpisującego się: Dzielnica Śródmieście Ulica Blacharska Nr. 22.

Zakład naukowy, w którym zapisujący się przebywał w poprzedzającym okresie (semestrze lub trymestrze): Pryw. Gimn.-Hum. Męski I. T. S. H. i Gr. we Lwowie

Pobiera lub pobierał (a) stypendjum rocznie ... z fundacji ...
nadane dekretem ...
z dnia ... 192... L.

Stosunek do służby wojskowej

Wpisuje się jako student Wydziału matem.-przyr. na podstawie: świadectwa dojrzałości gimnazjum w Lwowie [Pryw. Gimn.-Hum. Męski I. T. S. H. i Gr. we Lwowie]
(nowowpisujący się) z dnia 18 czerwca L. 23
(już imatrykulowany) karty imatrykulacyjnej z dnia ... L. ...

Wymienienie Akad. Towarzystwa samopomocowego, do którego należy wpisujący się.
(Rubrykę tę należy wypełnić w myśl uchwały Senatu Akad. z dnia 8 czerwca 1923)

III. trymestr
Potwierdzenie

Fig. 1. Photo of the first page of Rubin Schatten's entry card for the first year of studies at the Jan Kazimierz University, September 26, 1929. *Photo from [Sc5]*

At that time, Schatten lived with his parents in the center of Lwów at Blacharska street 22, he was of Jewish faith, his mother tongue was Polish, and he had Polish citizenship (see [Sc4]).

In the period 1930–1931 R. Schatten was a board member of the Society of Jewish Students of Philosophy of Jan Kazimierz University in Lwów (see [Sc13, p. 23]). The curator of the Society in the years 1924–1939 was Prof. Hugo Steinhaus, who was also a supporting member of the Society.

At Jan Kazimierz University, lectures and exercises in which he had the opportunity to listen to or participate in years, and other subjects

were as follows (in alphabetical order of the lecturers; tr = trisemester (approximately 60 working days) or trimesters, h/w = hours per week – see [Sc5]):

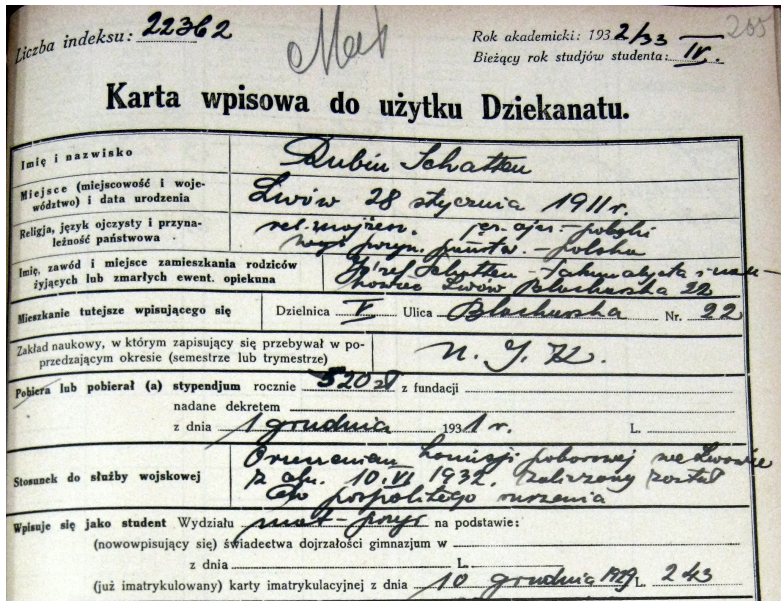
- 1929/1930: S. Banach, *Differential and integral calculus* (tr I - III, 4h/w) + exercises (tr I - III, 2h/w); S. Kaczmarz, *Analytic geometry* (tr II, III, 2h/w); S. Ruziewicz, *Introduction to higher algebra* (tr I, 2h/w), *Elliptic functions* (tr II, 2h/w), *Infinite operations* (tr III, 2h/w); J. Schauder, *Analytic functions* (tr I, II, 2h/w); E. Żyliński, *Introduction to number theory* (tr I, 2h/w), *Algebraic numbers* (tr II, III, 2h/w).

  K. Ajdukiewicz, The main principles of philosophical sciences (tr I, II, 4h/w); S. Loria, Experimental physics: mechanics and heat (tr I, 5h/w), electricity and magnetism (tr II, 5h/w), optics (tr III, 4h/w), Theory of physical measurements and error evaluation observations (tr III, 1h/w); Th. Reczyński, Thermodynamic radiation of solids (tr I, 2h/w); Z. Weyberg, Geometric crystallography (tr I - III, 3h/w).
- 1930/1931: S. Banach, *Higher analysis* (tr I - III, 4h/w) + exercises (tr I, II, 2h/w), it Lower mathematical seminar (tr I - III, 2h/w); S. Ruziewicz, *Power series* (tr I, II, 3h/w), *Dirichlet series* (tr III, 3h/w), Exercises from analytical geometry (tr. I - III, 1h/w); H. Steinhaus, *Calculus of probability* (tr II, 1h/w; tr III, 2h/w); E. Żyliński, *Differential equations* (tr I - II, 3h/w), *Calculus of variations* (tr III, 3h/w), *Algebraic numbers* (tr I, 2h/w), *Set theory* (tr II, III, 2h/w).

  S. Banach, General mechanics (tr I - III, 3h/w) + exercises (tr I, 2h/w), Dynamics of a rigid body (tr I - III, 1h/w); S. Loria, Exercises in studio I (physical) (tr 1 - III, 3h/w).

Unfortunately, in the archives in Lwów [Sc5], entries for the academic year 1931/1932 from the letters M to Z have not been preserved. However, we know which courses were available. Schatten had to pass some of them, especially the lectures.

- 1931/1932: S. Banach, *Theory of functions of real variables* (tr I - III, 3h/w), *Theory of functional operations* (tr I - III, 2h/w), *Seminar on the theory of functions of real variables* (tr I - III, 2h/w), *Seminar from a more recent theory of operations* (tr I - III, 2h/w); L. Chwistek, *Introduction to symbolic logic* (tr I, 1h/w), *Symbolic logic* (tr II, III, 1h/w); J. Schauder, *Topology* (tr I - III, 2h/w), E. Żyliński, *Higher algebra* (tr I - III, 2h/w), *Analytic functions* (tr I - III, 2h/w).
- 1932/1933: S. Banach, *Theory of functions of real variables* (tr I - III, 4h/w); S. Ruziewicz, *Analysis* (tr I - II, 3h/w), *Infinite products and continued fractions* (tr I, 2h/w), *Set theory* (tr II, 2h/w); H. Steinhaus,


Liczba indeksu: 22362

Rok akademicki: 1932/33

Bieżący rok studjów studenta: IV.

Karta wpisowa do użytku Dziekanatu.

Imię i nazwisko: Rubin Schatten

Miejsce (miejscowość i województwo) i data urodzenia: Lwów 28 stycznia 1911 r.

Religja, język ojczysty i przynależność państwowa: rel. mojż. jęz. ojcz. polski narod. pochodz. państw. polska

Imię, zawód i miejsce zamieszkania rodziców żyjących lub zmarłych ewent. opiekuna: Józef Schatten - tłumacz przysięgły Lwów Batorego 22

Mieszkanie tutejsze wpisującego się: Dzielnica I Ulica Batorego Nr. 22

Zakład naukowy, w którym zapisujący się przebywał w poprzedzającym okresie (semestrze lub trymestrze): U. J. K.

Pobiera lub pobierał (a) stypendjum rocznie 520 zł z fundacji ...

nadane dekretem ...

z dnia 1 grudnia 1931 r. L. ...

Stosunek do służby wojskowej: Orzeczeniem komisji poborowej we Lwowie z dn. 10.VI.1932. Zaliczony został do pospolitego ruszenia

Wpisuje się jako student Wydziału mat.-przyr. na podstawie:

(nowowpisujący się) świadectwa dojrzałości gimnazjum w ...

z dnia ... L. ...

(już imatrykulowany) karty imatrykulacyjnej z dnia 10 grudnia 1931 L. 243

Fig. 2. Photo of the first page of Rubin Schatten's card, student of fourth year at the UJK from 7 September 1932. *Photo from [Sc5]*

Theory of analytic functions (tr I - III, 3h/w); E. Żyliński, *Differential and integral calculus* (tr I - III, 5h/w).

E. Rybka, General Astronomy and Specular Astronomy (tr I - III, 2h/w).

We see in Fig. 2 that from 1 December 1931, Schatten received a scholarship of 520 zlotys a year. Moreover, by the decision of the Recruitment Committee in Lwów of 10 June 1932 he was recognized as fit for military service.

In 1933, Rubin Schatten wrote his master's thesis *Przestrzenie abstrakcyjne* [*Abstract spaces*] under the supervision of Stefan Banach (see Fig. 3). He submitted it for evaluation on 2 November 1933.

At the beginning of the twentieth century, several non-equivalent definitions of abstract spaces were given. Many of them were very successful, let us mention only: metric spaces (Fréchet 1906), Hausdorff topological spaces (Hausdorff 1914), axiomatics of general topological spaces (Kuratowski 1922).

In the introduction (see Fig. 4) to the master thesis by Schatten [S1] we read that the aim of the work was to provide theories of the simplest of abstract spaces by an appropriate selection of axioms, i.e., a set of any elements to which certain assumptions (certainties) are given. Schatten

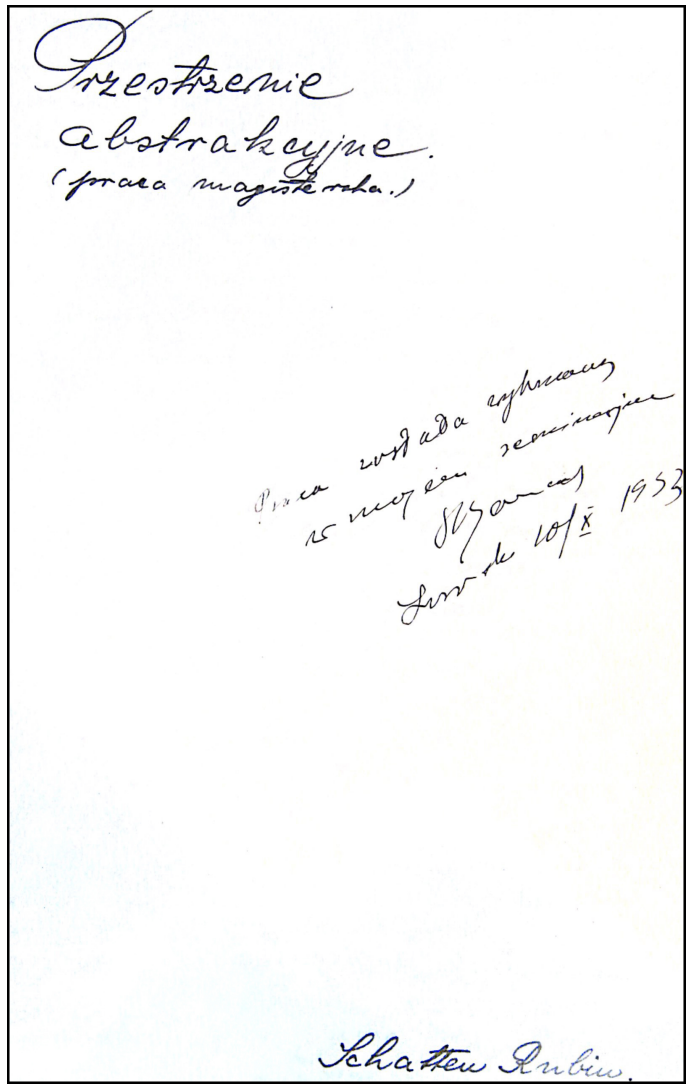
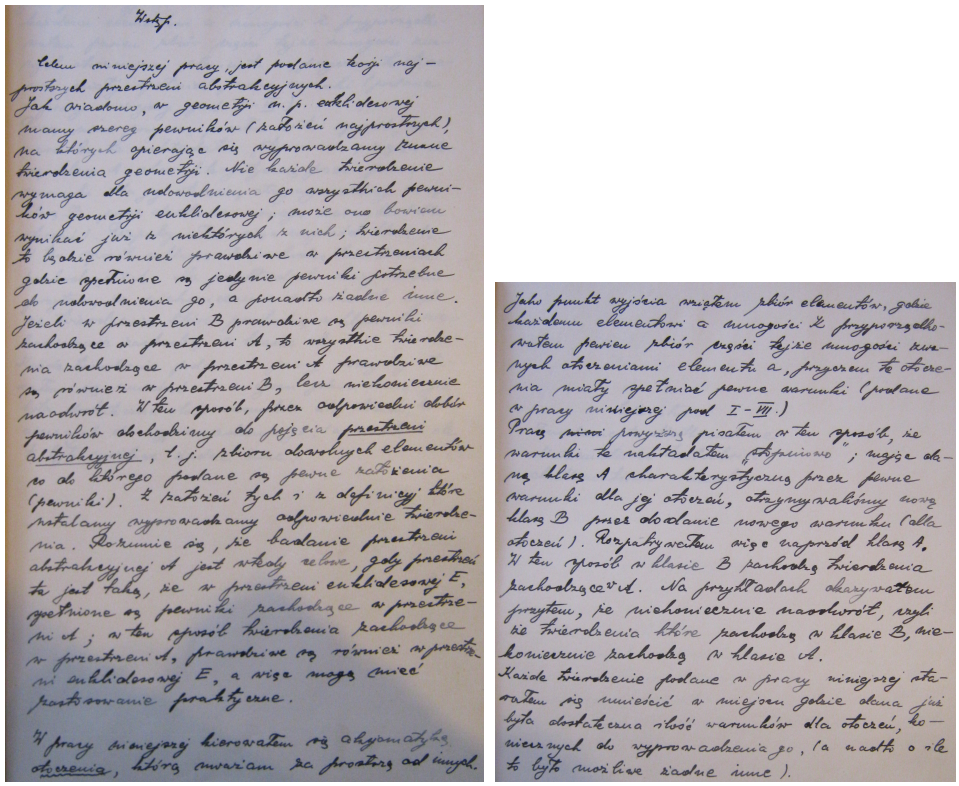


Fig. 3. Photo of the title page of Rubin Schatten's MA thesis *Przestrzenie abstrakcyjne* [*Abstract spaces*] with Banach annotation "The work was done in my seminar" dated 10 October 1933. *Photo from [Sc5]*

uses the *neighborhood* axiomatics, which he considers to be simpler than others. As a starting point, he takes a set of elements, where he assigns to each element a of the set K some set of subsets of this set called the neighborhood of an element a, where these neighborhoods have met some of the seven conditions (in the paper, these are conditions I – VII) that characterize a given class of spaces.

So we have Fréchet classes (V), classes (Z), classes (H) of Fréchet,



(a) The page 1 of the introduction. **(b)** The page 2 of the introduction.

Fig. 4. From the Schatten's master thesis. *Photo from [Sc5]*

classes (H) of Fréchet satisfying the stronger condition countability, Hausdorff topological spaces satisfying the weak axiom of countability condition, Fréchet normal (H) classes and metric spaces (Fig. 5).

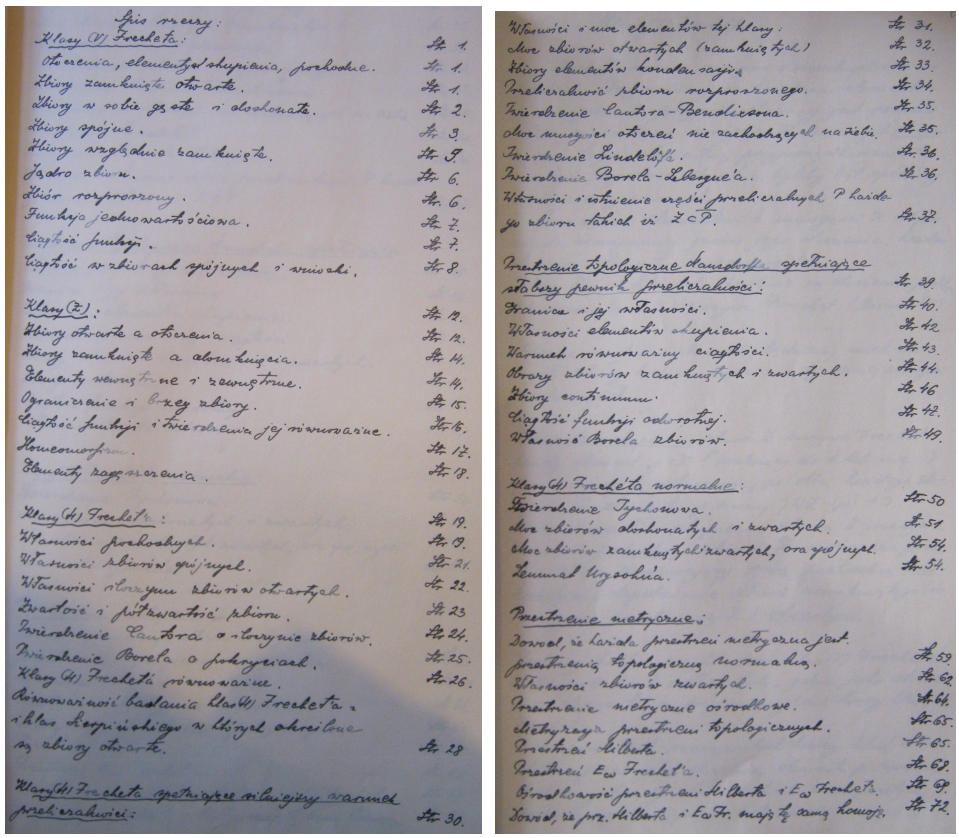
Schatten in his master thesis [S1] considers the Fréchet space s (as Fréchet, denotes it by E_ω) of all real sequences with a metric

$$\rho((x_n)_{n=1}^{\infty},(y_n)_{n=1}^{\infty})=\sum_{n=1}^{\infty}\frac{|x_n-y_n|}{n!\,(1+|x_n-y_n|)}.$$

This space, which is just a countable Cartesian product of the real lines $\mathbb{R}$, is separable, complete, and is a Fréchet space, i.e. a metrizable, complete and locally convex linear space ([6, p. 35]). Schatten first shows that the space s is homeomorphic to some subset of the Hilbert space l^2, then shows the homeomorphism of the Hilbert space l^2 with some subset of the Fréchet space s. His master's thesis [S1] ends with the following theorem (page 72):

The Hilbert space l^2 and the Fréchet space s of all sequences have the same homoeia.

This theorem was formulated and proved for the first time by Fréchet [11] in 1924. It is also proved in Fréchet's book [12, p. 84] from 1928 and in Sierpiński's book [30, p. 128] also from 1928.



(a) The page 1. **(b)** The page 2.

Fig. 5. The contents of Schatten's thesis. *Photo from [Sc5]*

Let us recall the concept of *homoeia* (homöie in German, homoja in Polish), which is also sometimes called a dimension in the sense of Fréchet introduced by him in 1910 in his paper [10] under the name "type of dimension". Let A and B be two topological spaces. If A is homeomorphic with some subset of B and B is homeomorphic with some subset of A, then we say that the spaces A and B have the same homoeia.

The term homoeia (homöie) was introduced in 1911 by P. Mahlo[1] in [22], where the Fréchet error from the paper [10] (see [12, p. 39], [13, p. 76]) is corrected.

This term is also mentioned in Sierpiński [30, p. 29] and in his next book [31, pp. 104–105], and discussed in more detail in the book [32, pp. 101–104] from 1947 and in its subsequent edition from 1965 (p. 100–103). In addition, we find the term homoeia and its properties can be found in Duda's book [8, pp. 56–58]. If the spaces are homeomorphic, then they have the same homoeia. Conversely, however, this is not the case. Spaces having the same homoeia need not be homeomorphic. Nevertheless, as Banach proved in [2, Corollary 2′], if two spaces A and B have the same homoeia, then each is the union of two disjoint subsets, $A = A_1 \cup A_2, B = B_1 \cup B_2$ such that A_k and B_k are homeomorphic, $k = 1.2$. Moreover, there are incomparable homoeias, in other words, not every two spaces can be compared using homoeia.

On the relationship between Fréchet's type of dimension and Urysohn–Menger's theory of dimension, see [18, Chapter V, §7]. It is worth noting that in [12] on p. 94, Fréchet gives a list of eleven function spaces having the same homoeia and on page 95 asks which of these spaces are not homeomorphic. This program, in particular, contains the question whether the Hilbert space l^2 and the Fréchet space s, which is the countable product of the lines $\mathbb{R}$, are they homeomorphic?

Sierpiński also informs in [30, p. 128] about this problem as an open problem. Banach's ascertainment in [3, p. 242, Chapter XI, §4; in the reprinted version - p. 212] raised a question of whether all infinite-dimensional separable Banach spaces are homeomorphic (1932).

The last sentence in Schatten's thesis [S1] is a statement on page 73:

> *The question whether Hilbert space and Fréchet space E_ω are homeomorphic was recently solved in the negative by Mr. Mazur.*

In Banach's book [3, p. 233, Chapter IV, §1; in the reprinted version - p. 206] we also find the statement that S. Mazur showed that the Fréchet space of all sequences s is not homeomorphic to any separable Banach space (see also [1, p. 515] and [20, p. 218]). Both of these statements turned out to be untrue.

[1]Paul Mahlo (28 July 1883–20 August 1971), German mathematician, specialist in set theory, published only in the years 1905–1928. You can read more about him in [14].

Schatten's master's thesis was assessed on 20 November 1933 by Banach and Steinhaus as very good. This review, if it existed, has not survived in the archives of the University of Lwów. That is why we cannot quote the opinion of Banach or Steinhaus on the originality of Schatten's thesis.

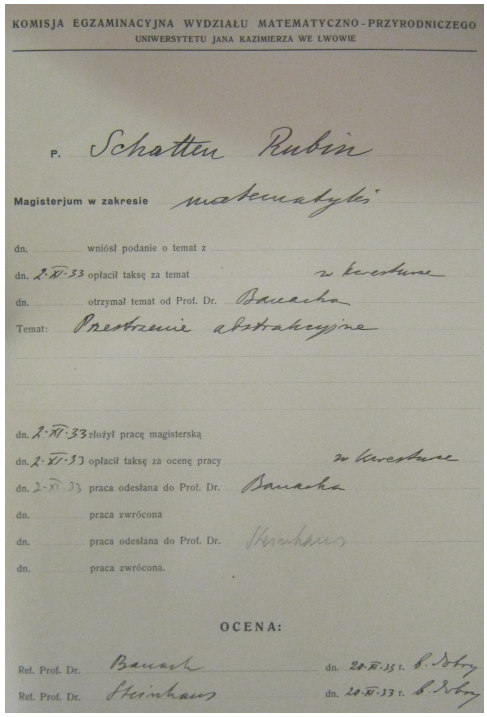
Schatten exams for the degree of Master of Philosophy in mathematics in the period June 1930–November 1933 were as follows (in chronological order with ratings (see Fig. 6b)):

> Differential and integral calculus with an introduction to analysis (24 VI 1930, 5), Principles of higher algebra with elements of number theory (10 XII 1930, 5), Experimental physics (16 XI 1931, 5), Analytical geometry (14 III 1932, 4), Logic (14 VI 1932, 5), Higher analysis (XII 1932, 4), The main principles of the philosophical sciences (17 III 1933, 4), Theoretical mechanics (III 1933, 5), The theory of functions of a real variable (15 XI 1933, 5), Master thesis *Abstract spaces* (20 XI 1933, 5).

We see that Schatten's thesis was related to one of two problems, the first of which was posed in Fréchet's 1928 book (see [12, p. 94–96]): Is l^2 homeomorphic to s? and the second inspired by Fréchet and a statement from Banach's 1932 book (see [3, p. 242]): Are all infinite-dimensional separable Banach spaces homeomorphic to the Hilbert space l^2?

It took more than thirty years to solve these problems and marked the beginning of an infinite-dimensional topology systematically presented in Bessaga and Pełczyński's book [6] (see also [20]). The answer to Mazur's statement is given by the Kadec–Anderson theorem (Kadec (1966) for Banach spaces, Anderson (1966) for Fréchet spaces): Every separable, infinite-dimensional Fréchet space, i.e. a metrizable and complete space, is homeomorphic to Hilbert space l^2. Kadec's work [19] was presented by L. V. Kantorowicz on 1 November 1965, and Anderson's article [1] was communicated by V. Klee on 24 January 1966. Anderson on the first page of his paper [1] refers to the Kadec paper [19]. H. Toruńczyk (1981) extended this result to a nonseparable case.

In [19] Kadec proved that all infinite-dimensional, separable Banach spaces are homeomorphic. On the other hand, Anderson in [1] proved that the Hilbert space l^2 is homeomorphic to the space s, which is a countable Cartesian product of the lines $\mathbb{R}$. The final result now follows from C. Bessaga's and A. Pełczyński theorem from 1962 ([5, Remark 1], [4, p. 275, §9.2], [6, p. 231]): While the hypothesis that all infinite-dimensional separable Banach spaces are homeomorphic to l^2 is true,


KOMISJA EGZAMINACYJNA WYDZIAŁU MATEMATYCZNO-PRZYRODNICZEGO
UNIWERSYTETU JANA KAZIMIERZA WE LWOWIE

P. Schatten Rubin

Magisterjum w zakresie matematyki

dn. wniósł podanie o temat z

dn. 2·XI·33 opłacił taksę za temat w kwestura

dn. otrzymał temat od Prof. Dr. Banacha

Temat: Przestrzenie abstrakcyjne

dn. 2·XI·33 złożył pracę magisterską

dn. 2·XI·33 opłacił taksę za ocenę pracy w kwestura

dn. 2·XI·33 praca odesłana do Prof. Dr. Banacha

dn. praca zwrócona

dn. praca odesłana do Prof. Dr. Steinhaus

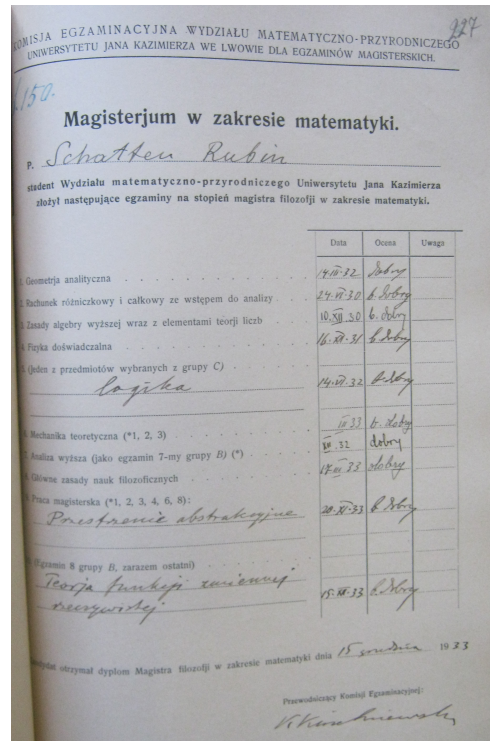
dn. praca zwrócona.

OCENA:

Ref. Prof. Dr. Banach dn. 20·XI·33 r. b. dobry

Ref. Prof. Dr. Steinhaus dn. 20·XI·33 r. b. dobry

(a) Photo of the master's thesis evaluation report from 20 November 1933. *Photo from [Sc5]*


KOMISJA EGZAMINACYJNA WYDZIAŁU MATEMATYCZNO-PRZYRODNICZEGO
UNIWERSYTETU JANA KAZIMIERZA WE LWOWIE DLA EGZAMINÓW MAGISTERSKICH.

227

150.

Magisterjum w zakresie matematyki.

P. Schatten Rubin

student Wydziału matematyczno-przyrodniczego Uniwersytetu Jana Kazimierza złożył następujące egzaminy na stopień magistra filozofji w zakresie matematyki.

	Data	Ocena	Uwaga
1. Geometrja analityczna	14·III·32	dobry	
2. Rachunek różniczkowy i całkowy ze wstępem do analizy	24·III·30	b. dobry	
3. Zasady algebry wyższej wraz z elementami teorji liczb	10·III·30	b. dobry	
4. Fizyka doświadczalna	16·III·31	b. dobry	
5. (Jeden z przedmiotów wybranych z grupy C) logika	14·III·32	dobry	
6. Mechanika teoretyczna (*1, 2, 3)	III 33	b. dobry	
7. Analiza wyższa (jako egzamin 7-my grupy B) (*)	III 32	dobry	
8. Główne zasady nauk filozoficznych	17·III·33	dobry	
9. Praca magisterska (*1, 2, 3, 4, 6, 8): Przestrenie abstrakcyjne	20·XI·33	b. dobry	
10. (Egzamin 8 grupy B, zarazem ostatni) Teorja funkcji zmiennej rzeczywistej.	15·XII·33	b. dobry	

Kandydat otrzymał dyplom Magistra filozofji w zakresie matematyki dnia 15 grudnia 1933

Przewodniczący Komisji Egzaminacyjnej:

(b) Photo of Rubin Schatten's master's exam protocol with information about his Master of Philosophy diploma in mathematics from 15 December 1933. *Photo from [Sc5]*

Fig. 6. Schatten's Master of Philosophy diploma documents

all infinite-dimensional separable Fréchet spaces not isomorphic to s are homeomorphic to l^2. In [4] l^2 is replaced with l^1, but as we know from Mazur's paper [25, str. 85], spaces l^1 and l^p are homeomorphic to any $p \geq 1$. So the above formulation with l^2 is correct.

From 15 December 1933 (see photos on Fig. 6), he was a Master of Philosophy in the field of mathematics. Unfortunately, we do not know what Rubin Schatten was doing in the years 1934–1936. Has he been to Lwów?

What we do know from [Sc6] is his arrival on the 12 September 1936, on the ship S.S. Piłsudski to New York in USA. Rubin Schatten was sailing from Gdynia, Poland, on the 3rd of September 1936 and arriving at the port of New York on the 12th of September 1936. The parents' address of this passenger is denoted "Lwow, Blacharska 22", and the

destination address is "New York University, Washington Square N. Y. (School of Commerce)." The answer to the question "Whether an alien intends to return to the country whence he came ..." is denoted "yes." Passage was paid by his father. Maybe R. Schatten returned to Poland and arrived in the USA in 1938.

However, there is information that Rubin Schatten was elected as a nominee for the Institutional Memberships of Columbia University of AMS in December 1937. Therefore, he should have worked at the university then. In fact, the forty-fourth Annual Meeting of the American Mathematical Society was held at Butler University, Indianapolis, Indiana, from Tuesday to Thursday, December 28-30, 1937 (see [Sc7]). On page 158 there is confirmation: "It was reported that the following had been elected as nominees on the Institutional Memberships of the various institutions and departments: ... Columbia University: Rubin Schatten and Walter Moakler Wynne."

In [Sc8] it is written (in column 22) that he was in the United States from 1936 to 1938 and will joint a friend Julius Siegel (see also [Sc11]). For some reason, Schatten was in Havana, Cuba, because on 2 February 1938, he arrived from Havana on the vessel SS Florida to Miami (see [Sc8] and [Sc9]).

In [Sc10] we find the following information about Rubin Schatten. He now residing at 431 Riverside Drive, New York and is a student of 27 years old. His personal description is: complexion-ruddy, color of eyes-blue, color of hair-brown, height-5 feet 7 (means 170 centimeters), weight-140 pounds (means 63 kilograms) and is not married. There is also such a description

> *I emigrated to the United States of America from Havana, Cuba; my lawful entry for permanent residence in the United States was at Miami, Florida; under the name Rubin Schatten, on February 2, 1938 on the vessel SS Florida.*

What we do know from [Sc17] is his arrival on 2 February 1938, on the ship S.S. Florida to Miami in the USA. He came alone and was 27 at the time. He enrolled at Columbia University in New York City and received his Master of Arts degree in December 1938 (promoted in 1939). From 1940 to 1942, he lived at 500 Riverside Drive in New York City.

On 25 October 25 1941, he attended the 381st meeting of the American Mathematical Society (AMS), where he gave a lecture *On the direct product of Banach spaces* as Rubin Schatten. In December 1942, he obtained a doctorate for his doctoral dissertation with the above title

at Columbia University in New York, but as Robert Schatten. The supervisor was Francis J. Murray[2].

The promotion took place in 1943 and in the same year the papers [S2], [S3] were published, the content of which consisted of a doctoral thesis.

He briefly taught at Columbia University's College of Pharmacy before being drafted into the U.S. Army, where he served from 24 July 1942 to 21 August 1943. During military training at Fort Benning, Georgia, he was injured and discharged from service as a result of a lifelong back injury.

On 24 April 1943, at the 395th AMS meeting at Hunter College, he gave a lecture *On certain properties of crossnorms*. Schatten became a naturalized U.S. citizen on 24 May 1943.

In the academic year 1943/1944 he was an assistant professor at the University of Vermont in Burlington. Receiving a two-year National Research Council scholarship, he studied for a year at the Institute of Advanced Study in Princeton: (9/1944–6/1945) and for almost the entire next year at Yale University in New Haven: (9/ 1945–6/1946). During this time he collaborated with John von Neumann and Nelson Dunford, during which periods joint works were produced with them: [S5], [S6] and [S8].

In 1946, he began a long-term cooperation with the University of Kansas, first as associate professor until 1952, then as full professor until 1961 (see [Sc1], [Sc14]), with breaks for repeated stays at the Institute of Advanced Studies (6/1950–1/1951 and 8/1952–9/1953). At AMS meetings he presented lectures, among others: on 10 April 1948 he gave the lecture *A characterization of sphere*, and at the 55th Meeting of Mathematicians at Ohio State University, Columbus (28–30 December 1948), attended by over 500 people, he gave a lecture *Closing-up of sequence spaces* (see [Sc19, p. 289]), which was published in [S10]. From [Sc19] we learn that the congress was attended, besides Schatten, by other mathematicians related to Poland: e.g. Nachman Aronszajn, Stefan Bergman, Zygmunt W. Birnbaum, Samuel Eilenberg, Mark Kac, Otton Marcin Nikodym, Alfred Tarski and Antoni Zygmund.

[2]Francis Joseph Murray (3 February 1911–15 March 1996), American mathematician, known for his papers in functional analysis, including a fundamental paper (with John von Neumann [26]) on rings of operators, which later became known as von Neumann algebras. He received his doctorate from Columbia University in 1936, where he also began working and was promoted to professor in 1949. In the 1960s–1980s, he taught at Duke University.

Let us note that between 1951 and 1978, Nachman Aronszajn[3] worked at the University of Kansas, he was a Polish Jew and, like Schatten, lost his entire family in World War II. However, we do not know what their relationship was like.

In the years 1960–1961, Schatten spent a year at the University of Southern California in Los Angeles as a visiting professor, and in the years 1961–1962 he taught at the State University of New York at Stony Brook. From 1962 until the end of his life, he was a professor at Hunter College in New York (on the photo Fig. 7).

In [Sc1, str. 319] we read that

> *Professor Schatten developed a real fondness for Lawrence and an attachement to the University of Kansas, but eventually he sought the big city: resigned in 1961, and he has been a member of the faculty of Hunter College in New York since 1962.*

Every year he gave a lecture on "Advanced Calculus" there. In Schatten's short biography [Sc2, 21, p. 138], one of his former PhD students Elliot Cheney Jr. states:

> *To his former students, Schatten will be remembered as a dedicated teacher who was genuinely concerned with the intellectual development of his students. They will certainly not forget his unique style of lecturing. He always spoke without a book or notes, and rarely used the blackboard. His lectures were extremely clear and well-organized; he never lost his way in complicated arguments. The pace was such that the students could (and were expected to) take notes verbatim; if they did so, their notes would read like a polished book, except for some linguistic idiosyncracies such as, "Given is a set...". He left nothing to chance in his dictation; for example, he invariably ended an argument with "This concludes the proof."*

[3]Nachman Aronszajn (26 July 1907–5 February 1980), Polish-American mathematician born in Warsaw and educated in Poland. He obtained his doctorate at the University of Warsaw under supervision of S. Mazurkiewicz (1929) and the second time at the University of Paris under supervision of M. Fréchet (1935). He spent his professional life as a mathematician mainly in France (1930–1940 and 1945–1948), England (1940–1945) and the United States (after 1948, including Kansas University in Lawrence: 1951–1978). He specialized in mathematical analysis. You can read more about it in the article [33].



Fig. 7. 1975. Robert Schatten in a dark jacket with a group of students from Hunter College

In the years 1964–1972 he was also a member of the doctoral faculty at the Graduate School of the City University of New York, where, among others, gave a lecture on Hilbert spaces.

As far as we know, Schatten has supervised three PhDs in mathematics. In 1957, at the University of Kansas, it was Elliott Cheney Jr.[4] with a doctoral dissertation "On gauge functions". The next two doctorates, under the supervision of Schatten, were completed in April 1968 at City University in New York City. They were: Peter Falley, "Ideals of operators", 26 April 1968, 59 pages, and Charles D. Masiello, "The average of a gauge", 26 April 1968, 27 pages. His paper was published in [24].

In the descriptions [Sc2], [Sc3] and [Sc16], we read that Schatten

[4]Elliott Ward Cheney Jr. (28 June 1929–13 July 2016), American mathematician, one of the pioneers of numerical analysis and related problems of approximation theory. His 1966 book, *An Introduction to Approximation Theory*, is "highly respected and well known". The book [21], co-authored with William Allan Light, is *dedicated to the memory of Robert Schatten (1911–1977), who did much of the pioneering work in the theory of tensor products of Banach spaces.* Also included in this book is a short biography of Schatten [Sc2].

hated noise, especially when it disturbed his sleep. Bachelor life also had its pitfalls, such as having to contend with laundromats that insisted on ironing socks. He kept his unpublished mathematical research in a bank safe.

Robert Schatten died on 26 August 1977 in New York. He had no living heirs at the time of his death; he lost his entire family during World War II (see [Sc2], [Sc3]). In [Sc15] we read that members of Schatten's family died in concentration camps in Europe, he himself did not leave anyone close to him. Schatten died at The Mount Sinai Hospital in Manhattan, which was the first Jewish hospital in New York City.

2. Scientific achievements of Robert Schatten. Schatten dealt with Banach spaces and linear transformations in Hilbert spaces, among other things. Schatten's main mathematical achievement was initiating the study of tensor products of Banach spaces. The concepts of "cross norms", associated norms, largest, smallest, and uniform "cross norms", which are mostly derived from him, or at least were first carefully studied in his articles. His name has been immortalized in Schatten classes (see [7], [27], [29]).

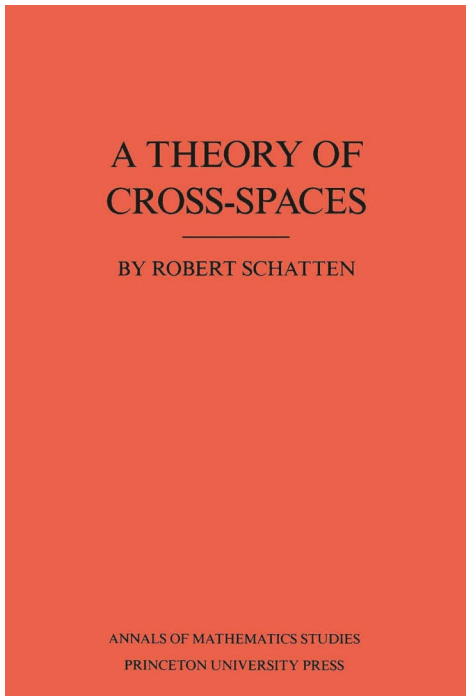
He wrote 13 mathematical papers and two books [S9], [S12]. The first book on normed tensor products was the 1950 monograph [S9], which summarized the state of the art at the time. The second book from 1960 [S12] is about operator ideals on Hilbert spaces.

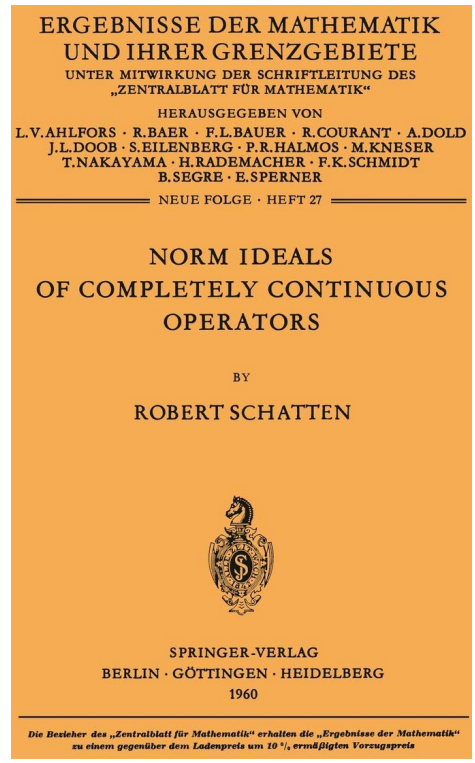
2.1. Tensor products of Banach spaces. Tensor products first appeared in Functional Analysis in the late 1930s in Murray and John von Neumann's [26] paper on rings of operators on Hilbert spaces. The first systematic studies of tensor products in Banach spaces were initiated in the forties: Schatten (1943, 1946 in [S2], [S3], [S4]), Dunford–Schatten (1946 in [S5]), and Schatten–von Neumann (1946, 1948 in [S6], [S8]).

Schatten's main achievement in this field was the initiation of research on tensor products of Banach spaces. The concepts of *crossnorm, associated crossnorm, greatest crossnorm, reflexive crossnorm, uniform crossnorm* are mostly derived from him, or at least were first thoroughly explored in his papers.

The first book on normed tensor products was R. Schatten's 1950 monograph [S9], which summarized the state-of-the-art at the time (the cover is on photo Fig. 8).

In the book's acknowledgments (see [S9, p. 6]), Schatten admits





(a) The cover of Schatten's book [S9]. **(b)** The cover of Schatten's book [S12]

Fig. 8. The covers of Schatten's books

that his research with Prof. J. von Neumann in the years 1944–1946, i.e. during his stay at The Institute for Advanced Study, and the subsequent exchange of correspondence in the years 1946–1948 played an important role in the preparation of this book. The ideas contained herein come from both the author and Professor von Neumann. Some of them were published, in a different form, in their joint works [S6] and [S8].

Both Schatten's papers from 1943–1948 and his book [S9] are pioneering studies. However, the breakthrough in tensor products was the work of Alexander Grothendieck[5] from the 1950s [16] and [17]

[5]Alexander Grothendieck (28 March 1928–13 November 2014), born in Berlin, a stateless person, and from 1939 a French mathematician living in France. One of the greatest mathematicians of the twentieth century. Initially, he dealt with functional analysis, especially the theory of Banach spaces and linear topological spaces and their tensor products. Creator of the modern foundations of algebraic geometry. Recipient of the Fields Medal in 1966. More about his life and mathematics is described in the article [28].

Grothendieck cites Schatten's book [S9] in his works: in [15] on pages 74, 76, 81 and 112; in [16] on pages 53, 75–77 and in [17] on pages 4, 28. However, he did not use a single Schatten result. It is likely that due to poor scientific communication in the postwar period, Grothendieck was unaware of Schatten's approach when he began his own research (see also [27, p. 373]).

As we know, Grothendieck stayed from 11 January 1955 for half a year at the University of Kansas, at the invitation of the aforementioned Nachman Aronszajn, who also worked in mathematical analysis. Maybe Schatten and Grothendieck were talking to each other. It is known, however, that while in Kansas in 1955, Grothendieck abandoned analysis and began research in algebraic geometry.

2.2. Schatten classes S_p. In [S9] Schatten defined the classes (spaces) S_p as a non-commutative analog of the spaces l^p. The structure of the S_p class resembles that of l^p.

Let H be a separable (complex) Hilbert space, and let $B(H)$ denote the space of linear bounded operators on H and $K(H)$ the closed subspace of all compact operators on H. For $1 \le p < \infty$, let $S_p = S_p(H)$ be the space of operators $T \in K(H)$ such that the *Schatten norm*

$$\|T\|_{S_p} = (\sum_{n=1}^{\infty} s_n(T)^p)^{1/p} < \infty,$$

where $s_1(T) \ge s_2(T) \ge \ldots \ge s_n(T) \ge \ldots \ge 0$ are the singular values of T, i.e. the eigenvalues of the Hermitian operator $|T| := (T^*T)^{1/2}$. It is not entirely obvious, but is true, that this a norm and that the classes of such operators $S_p(H)$ with Schatten norms are Banach spaces. They are also ideals in $B(H)$.

Note that the Schatten class $S_2(H)$ is, in fact, a Hilbert space of Hilbert–Schmidt operators and a Schatten class $S_1(H)$ is the space of trace class operators.

The Schatten norm satisfies a type of Hölder–Rogers[6] inequality:

$$\|ST\|_{S_1} \le \|S\|_{S_p}\|T\|_{S_{p'}} \quad \text{if} \quad S \in S_p, T \in S_{p'} \quad \text{and} \quad 1/p + 1/p' = 1,$$

where by definition $S_\infty = K(H)$.

Schatten classes $S_p(H)$ are separable; the set of finite rank operators is dense in $S_p(H)$. Moreover, if $1 \le p < q \le \infty$, then $S_p(H)$ is dense

[6]The classical Hölder inequality should historically be correctly called the Hölder–Rogers inequality (cf. [23]).

linear subspace of $S_q(H)$ and $\|T\|_{S_q} \leq \|T\|_{S_p}$ for all $T \in S_p$. Also $S_p(H)$ is isometrically isomorphic to the dual of $S_{p'}(H)$ if $1 < p < \infty$, and to the dual of $K(H)$ if $p = 1$.

2.3. Other papers by Schatten

We will discuss results from Schatten's papers that have not been previously cited.

In a paper [S7], published in 1947, Schatten showed that if X_0 is a closed subspace of a Banach space X, and there is a projection of X onto X_0 with bound 1, then the greatest crossnorm γ on the tensor product $X \otimes Y$ is an extension of the greatest crossnorm on $X_0 \otimes Y$ for any Banach space Y (see [S7, Lemma 2, p. 323]; see also [S9, Lemma 3.7, p. 55]).

In a paper [S10], published in 1950, Schatten discussed two methods for completing a linear sequence space c_{00} of real sequences $(u_1, u_2, \ldots)$ having only a finite number of non-zero terms, with respect to a standard "symmetric"" and normalized norm $\phi(u_1, u_2, \ldots)$. The first method (strong closure) consists in the construction of all sequences $(u_1, u_2, \ldots)$ for which $\lim_{n\to\infty} \phi(u_1, \ldots, u_n, 0, 0, \ldots) < \infty$; the second method is the Cantor–Méray completion of a metric space[7]. The Cantor–Méray completion is always included in the strong closure and examples are given in Schatten's paper which show that the two can be distinct.

In a paper [S11], published in 1957, Schatten proved the interesting result that the unit ball in the space $K(H)$ of compact operators on a separable infinite-dimensional Hilbert space H has no extreme points, and hence by the Krein–Milman theorem is not isomorphic to any conjugate space X^* of a Banach space X. Important tools in the proof was the so-called "polar representation" of a compact operator on H and methods of topological tensor products.

Schatten's 1962 paper [S13] initially resembles the approximation theorems of Weierstrass and Müntz, but the main part of this expository paper was a discussion on the non-aproximation theorem. Schatten proved in elementary and readable way result from [S5, p. 432] that the function $F\colon [0,1] \times [0,1] \to \mathbb{R}$ defined by

$$F(s,t) := \begin{cases} 1 & \text{if } 0 \leq s \leq t \leq 1, \\ 0 & \text{if } 0 \leq t < s \leq 1, \end{cases}$$

[7]The construction of the real numbers from the rationals via equivalence classes of Cauchy sequences is due to Georg Cantor (1872) and Charles Méray (1869). Of course, their method can be used to complete general metric spaces.

is not approximable in the norm $L^\infty([0,1]\times[0,1])$ by finite sums of "products, that is, does not lie in the closure of finite linear combinations of the form $\sum_{k=1}^{n} f_k(s)\,g_k(t)$ with $f_k, g_k \in L^\infty[0,1]$.

In a report [S14], published in 1962, Schatten considered a completely continuous operators on a complex Hilbert space H. He proved that every operator in the trace-class is necessarily in the Schmidt-class and every operator in the Schmidt-class is completely continuous

In a paper [S15], published in 1964, Schatten showed possibility to consider the concept of a trace-class of operators on general Banach spaces. He also proved some results for such operators.

In Schatten's last paper [S16] we have an elementary expository discussion of the Schauder basis problem and the approximation problem in Banach spaces. The paper was received by the editors on 15 March 1973 and was dedicated to Professor Beniamino Segre[8] for his 70th birthday. If a Banach space has a Schauder basis then it has the approximation property, but Per Enflo constructed a Banach space without the approximation property and published it in 1973 in [9]. Schatten was unaware of the existence of this counterexample.

In the book [Sc12, pp. 161–162], besides Schatten's publications [S4] – [S10] is mentioned *Work on a book on Spectral Theory. In progress.* It is unclear whether this text or part of it is in the book published as [S10].

Acknowledgments: We thank Professor Barry Cherkas who sent photo 7 to Ms Mayram Vulis and she shared it with us. Furthermore, we thank Prof. Jan Strelcyn of Paris for a very careful proofreading of our text, for a detailed analysis of Fréchet's results contained in the papers [10], [11] and his book [12], for information about the paper [20] and for details on the history of the proof of the Kadec–Anderson theorem. Additionally, the authors would like to thank one of the two referees for copies of documents [Sc6] – [Sc11], which reached us in February 2024 and which provides better information than that in [Sc4] about what happened to Schatten in the years 1936–1938. We also thank Marie-Claude Werquin from Paris for polishing the English language.

Author Contributions: All authors equally contributed to the conceptualization, methodology, formal analysis, investigation, and writing–original draft preparation.

Funding: This research received no external funding. Polish Mathematical Society remains neutral with respect to jurisdiction claims in published maps and institutional affiliations.

[8] Beniamino Segre (1903–1977), Italian mathematician. He was born and studied in Turin. He was a professor at the University of Bologna (1931–1938 and 1946–1973). In 1938, he lost his professorship as a result of the anti-Jewish laws enacted under the rule of Benito Mussolini. He spent the next 8 years in Great Britain (mainly at the University of Manchester). He is known as one of the main algebraic geometers of the first half of the 20th century and one of the founders of finite geometry.

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Rubin (Robert) Schatten (1911–1977)

Lech Maligranda i Yaroslav G. Prytula

Streszczenie. Rubin (Robert) Schatten był polsko-amerykańskim matematykiem zajmującym się głównie analizą funkcjonalną. W latach 1929–1933 studiował na Uniwersytecie Jana Kazimierza we Lwowie, a pracę magisterską ukończył u Stefana Banacha. Od września 1936 roku aż do śmierci w sierpniu 1977 roku przebywał w USA, gdzie uzyskał doktorat w 1943 roku i gdzie pracował na kilku uniwersytetach amerykańskich. Schatten opublikował 13 prac naukowych i dwie monografie z teorii operatorów. Był jednym z pionierów systematycznego badania iloczynów tensorowych przestrzeni Banacha. W matematyce jego nazwisko zostało uwiecznione w klasach operatorów Schattena. Zainteresowaliśmy się Schattenem ze względu na początki jego kariery we Lwowie oraz brak zdjęcia we wszelkich opisach dotyczących jego osoby. Przedstawiamy więc koleje jego życia, o których niestety wiemy bardzo mało. Po raz pierwszy pojawi się też jego zdjęcie.

Klasyfikacja tematyczna AMS (2020): 01A60, 01A70, 01A73.

Słowa kluczowe: Historia matematyki i matematyków, Robert Schatten, analiza funkcjonalna, iloczyn tensorowy, klasy Schattena.

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Communicated by: Małgorzata Terepeta

(Zgłoszona: 11 listopada 2023; Wersja końcowa: 16 luty 2024)